# Estimate of diffraction from Gaussian Beam in Li-Baker HFGW detector

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**Abstract.** Recent experimentation and speculation about the design of a sensitive detector for high-frequency gravitational waves (HFGW) has centered around a number of principles. Those detectors that have been built so far have not yet realized sensitivity sufficient to investigate the cosmic high-frequency relic gravitational wave background, analogous to the cosmic microwave background. A proposal for a more sensitive HFGW detector due to Baker and based upon a principle first enunciated by Li and co-workers has become known as the Li-Baker detector. Its possible design details are currently the subject of scientific debate. One significant aspect concerns the diffraction of microwave power from the intrinsic transmitter producing a microwave beam. This beam is required for interaction with the gravitational waves to be detected and diffraction will not be distinguishable from photons produced by interaction with the HFGW. This means that diffraction is potentially a source of shot noise at the microwave receivers and, if extreme, may also swamp the receivers. In this paper some estimates of this diffraction are obtained. These estimates show that the Li-Baker detector must be designed in such a way that the diffraction reaching the microwave receivers is reduced as far as possible by employing a suitable geometry and highly absorbent walls for the interaction volume.

**Keywords:** Detectors; Diffraction theory; Gravitational waves; High-frequency gravitational waves; HFGWs; Microwaves; Very-high-frequency gravitational waves; VHFGW
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## INTRODUCTION

Gravitational waves and their applications are now becoming an important emerging technology. The initial theoretical work of Einstein (1916), Forward and Miller (1967), and Romero and Dehnen (1981) laid the foundations that have supported subsequent experimentation. Hulse's (1994) and Taylor's (1994) confirmed observations of very-low-frequency gravitational waves of astronomical origin eliminated any remaining skepticism concerning their existence and nature. More recently, the properties of high-frequency gravitational waves (HFGWs, defined by Douglass and Braginsky (1979) as GWs having a frequency between 100kHz and 100MHz) and very-high-frequency gravitational waves (VHFGWs, defined by Douglass and Braginsky (1979) as GWs having a frequency between 100MHz and 100GHz) have been investigated theoretically. Typical proposed HFGW laboratory generators (Woods and Baker, 2005) are predicted to produce a GW signal that has a constant polarization angle, unlike a rotating binary star system in which the polarization angle rotates at twice the orbital frequency. For a binary star system in circular orbit, the GW emitted amplitude is constant whereas the predicted GW detected signal varies at the detector due to the shift in polarization. Woods (2005, 2006) has speculated upon applications of reported interactions of HFGWs with superconductors, which if confirmed would allow an entirely new field of HFGW optics to be developed.

There appears to be a fundamental problem regarding generation of GWs of all frequencies: enormous power input is required to produce even a tiny GW power output. At very low frequencies, only bodies of astronomical size are predicted to produce significant GWs while rotating. Subsequent development has included the large-scale LISA, LIGO, Virgo, GEO600 and other prototype projects for low-frequency gravitational wave detection. However, these highly-developed detectors have yet to confirm any observation of GWs directly, thus illustrating the low ratio of input to output power in typical celestial generators. Suggestions for alternative terrestrial generators have included mechanical devices (Halpern and Laurent, 1964; Forward and Miller, 1967), fundamental quantum and electromagnetic interactions (Gertsenshtein, 1962; Grishchuk, 2003), interactions involving superconductors (Fontana and Baker, 2003), electromagnetic actuators (Baker, 2004), piezoelectric actuators (Romero and Dehnen, 1981), acoustic resonators using magnetron excitation (Woods and Baker, 2005; Baker, Woods, and Li, 2006), and nuclear explosions (Chapline, Nuckolls, and Wood, 1974; Fontana, 2006). In most cases the generated VHFGW power is a minute fraction of the input power needed to create the required excitation. Only on using a nuclear interaction is the output power likely to be significant. However, this approach seems not to be a practical GW generation method that can potentially achieve wide market penetration, at least in the near future. The total quadrupole GW power radiated by a rotating rod is proportional to the sixth power of the rotational frequency (Baker, 2006) so there is also considerable advantage in working at higher frequencies. The most promising terrestrial VHFGW generator postulated so far appears to be an array of piezoelectric acoustic resonators arranged in a circle (Woods and Baker, 2005). The basic reason for this arrangement being so promising is that, as well as operating at the highest frequency readily accessible using current technology, the high quality factor (*Q*) of the resonating structure ensures a corresponding enhancement of the radiated power of the GW produced. Woods and Baker (2009) also proposed a possible HFGW generator operating at even higher frequency (equivalent to electromagnetic infra-red) exploiting intra-molecular bond vibrations.

Current developments in HFGW detectors are slightly more advanced. A number of instruments intended for detection of HFGW have been constructed. These include designs by Chincarini and Gemme (2003) using coupled microwave resonant cavities and by Cruise (2000) utilizing interactions in waveguides. Woods (2010) proposed a novel type of HFGW detector using an acoustic resonator and originally intended rather as a test of HFGW interaction with superconducting materials. Finally, a proposed detector design that has been the subject of some scientific debate recently is due to Baker (Baker, Stephenson and Li, 2008) and based upon a theoretical development originally due to Li (Li, Tang and Zhao, 1992). The basic concept of the Li-Baker detector (Li *et al.*, 2008) is a new synchro-resonance solution of the Einstein equations, related to but not the same as the Gertsenshtein effect (or inverse Gertsenshtein effect). In his classic paper, Gertsenshtein (1962) described how the non-linearity of Einstein's field equations required that in the presence of a static electric or magnetic field, electromagnetic (EM) wave propagation generated an associated coupled GW. In addition, if a static magnetic (or static electric) field is superimposed upon a GW propagating perpendicular to the field direction, the two will interact to generate an EM wave in the same direction as the GW (the inverse Gertsenshtein effect). The amplitude of the generated wave is, however, so small that practical exploitation of this effect is fraught with apparently insuperable difficulties. The new synchro-resonance solution of Einstein's field equations, by contrast, identifies a coupling between EM and gravitational waves (Li, Tang and Zhao, 1992) that arises according to the theory of relativity.

## THE LI-BAKER DETECTOR

A sketch of the complete Li-Baker detector is shown in Fig. 1. The detector has four major components: (*i*) Gaussian microwave beam (GB) directed along the +*z*-axis at the frequency of intended GW detection (Yariv, 1975) and typically in the GHz band, in the same direction as the HFGW to be detected; (*ii*) static magnetic field **B** directed along the *y*-axis; (*iii*) narrow-band reflector(s) which might be implemented as fractal membrane(s) (Wen *et al.*, 2002; Zhou *et al.*, 2003; Hou *et al.*, 2005), and (*iv*) high-sensitivity microwave receivers. Placing the reflector(s) outside the GB embodies the geometry specified in Fig. 4 of Stephenson (2009). In a slightly different arrangement shown in the inset of Fig. 1 (Baker, Stephenson and Li, 2008), the reflector(s) are placed within the GB and inclined at an angle to it, to reflect the signal photons onto the microwave receivers from within the GB (in a manner reminiscent of a classical Herschelian telescope). (Some alternative arrangements have also been suggested, such as using "microwave lenses" outside the GB to focus the PPF onto the receivers, or a double-sided reflector within the GB.) This detector adds the GB to the static field and GW required for demonstrating the inverse Gertsenshtein effect. In the Li-Baker detector, a first-order perturbative photon flux (PPF) will be generated in the *x*-direction (by contrast with the *z*-directed EM wave generated in the inverse Gertsenshtein effect). The design of the detector is then intended to isolate this PPF and to distinguish it from the superimposed EM wave, so that the GW is therefore detected by the presence of the PPF. The detection flux (PPF) is generated when the two waves (EM and GW) have the same frequency and a uniform phase difference along the z-axis (i.e., they are coherent in both space and time, the synchro-resonance condition). The PPF, comprising detection photons, is produced at the overlap of the static magnetic field, **B**, and the EM wave; the PPF travels in both directions on the *x*-axis, perpendicularly both to **B** (in the $y$-direction) and to the direction of the wave coherence. Therefore, the PPF can be intercepted by receivers located in regions in the detector (on the $x$-axis, well away from the $z$-axis) that are relatively noise-free since the photons from the EM wave (the background photon flux, or BPF) travel in the $z$-direction and, except for scattering, will not reach the receivers located on the $x$-axis. A resonant cavity can be used to enhance the amplitude of the resultant effect. The resultant conversion efficiency is much greater than from the inverse Gertsenshtein effect as exploited in previously proposed HFGW detectors. The proposed Li-Baker detector is sensitive to HFGW directed along the $+z$-axis, and the geometrical arrangement of the major components around this is the key to its operation. Using typical design parameters, Li *et al.* (2008) predict a strain sensitivity of $10^{-32}$ to $10^{-30}$ for the Li-Baker detector.

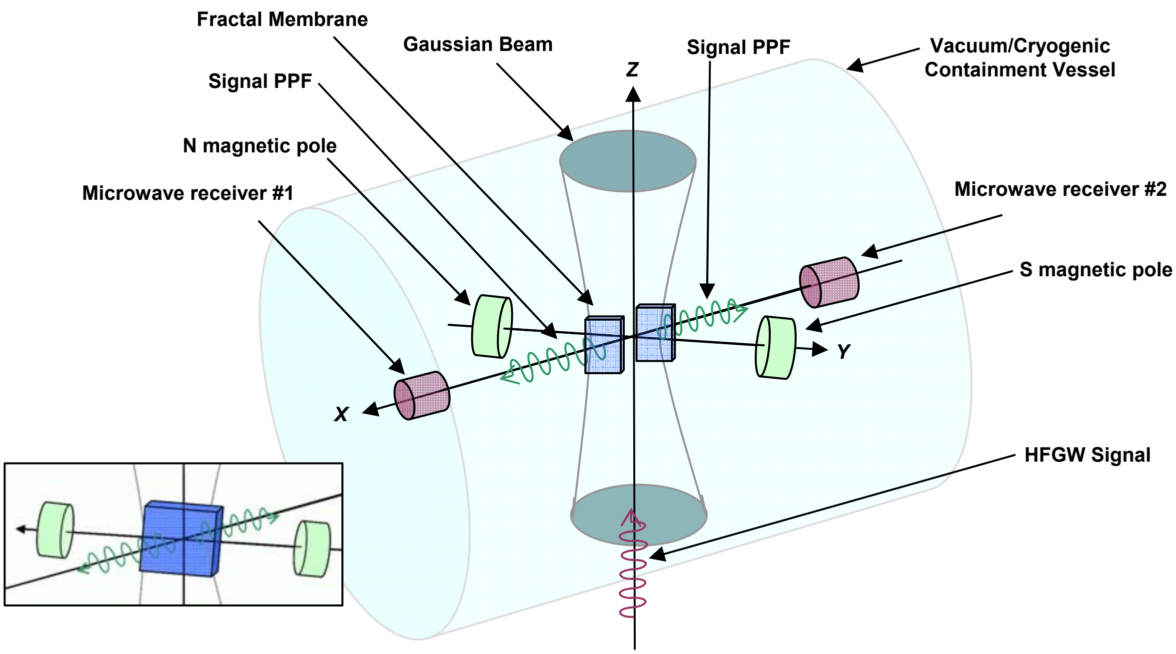


**FIGURE 1.** Basic layout of the Li-Baker HFGW Detector (Stephenson, 2009) using external reflectors. Inset, bottom left: modified layout using internal reflector, from Fig. 8 of Baker, Stephenson and Li (2008).

Potentially a significant problem in the design of this detector is that of diffraction of microwave power from the Gaussian beam. The microwave source for this beam is estimated to produce on the order of 1kW or more for a detector of reasonable sensitivity (Li *et al.*, 2008) and the microwave receivers must be low-noise amplifiers capable of detecting signals produced by interaction with HFGWs that are at or near the noise floor. This requires a high degree of isolation between the microwave source and the receivers. The objective of the present paper is to estimate the magnitude of the diffraction from the Gaussian beam that will be detected at the receivers, under free-space conditions. If too great, steps will need to be taken to minimize or reduce it to acceptable levels.

## INTRODUCTION TO DIFFRACTION THEORY

The concept of wave diffraction is central to the theory of physical optics. In fact, diffraction may be thought of as the mechanism by which an electromagnetic wave propagates through free space as well as how it interacts with obstacles and apertures. The first successful attempt to describe simple physical optical phenomena was the famous principle of Huygens, enunciated in the seventeenth century, which was developed in conjunction with the wave theory of light. Huygens's principle states that every point on an advancing wavefront (whether obstructed or not) is the center of a new disturbance and acts as a secondary source of a new system of spherical waves; and that, further, the resulting observed disturbance or advancing wavefront is the algebraic sum of all the Huygens secondary waves produced from every such source already traversed. Competing theories of the corpuscular or particle nature of light, unlike the successful Huygens's principle, were unable to account quantitatively for basic optical phenomena such as diffraction from apertures, wave patterns set up around obstructions, and refraction. As a result, the basic approach of Huygens (as developed by later theoreticians such as Fresnel and Airy in the nineteenth century) was widely regarded as largely correct and complete until the advances of quantum theory and the concept of wave-particle duality were required to explain more advanced effects.

For a wave system where the superposition of secondary wavelets to produce the observed disturbance is a good description of the behavior of the wave propagation, Fresnel introduced a simple emendation called the "obliquity factor" that improves the accuracy of Huygens's principle (Pedrotti, Pedrotti, and Pedrotti, 2007). Huygens implicitly assumed that secondary waves in the *forward* direction (i.e., for which the resolved wavevector component parallel to the incident wavevector points in *the same direction* as that incident wavevector) have uniform amplitude over the hemisphere of secondary propagation in the same direction as the incident beam, but that secondary waves in the *reverse* direction (i.e., for which the resolved wavevector component parallel to the incident wavevector points in *the opposite direction* to that incident wavevector) have zero amplitude. Fresnel showed that instead assuming that the amplitude is proportional to the obliquity factor $(1 + \cos\theta)/2$, where $\theta$ is the angle to the incident propagation vector, gave more accurate results. A comparison of the two obliquity factors is shown in Fig. 2. Huygens alone could not explain why the diffraction amplitude at large angle deviations is reduced over that at small angle deviations, or why it is possible experimentally for a small amount of energy to be diffracted rearwards, but Fresnel's obliquity factor predicts both these behaviors.

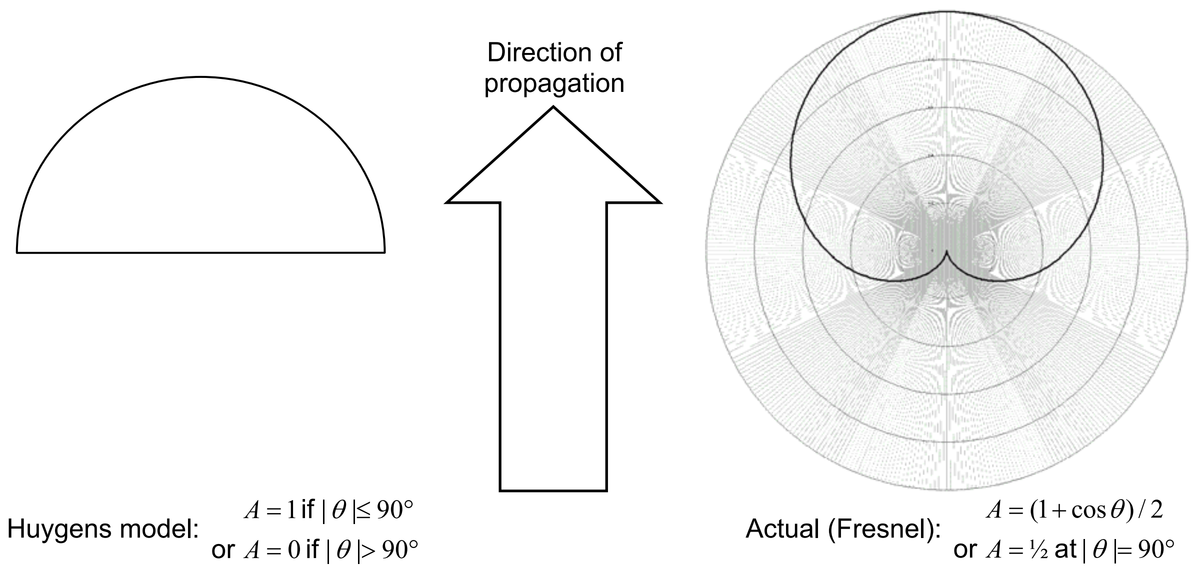


**FIGURE 2.** Comparison of Huygens and Fresnel Obliquity Factors (*A*).

Most treatments of wave diffraction (Saleh and Teich, 1991) start with an axial incident beam and calculate the diffracted power slightly off-axis using various approximations that generally are only good for small diffraction angles. Developments of the classical Airy approach, following this basic model, include those by Keller (1962) and by Sheppard and Hrynevych (1992). The situation in the Li-Baker detector is rather more extreme than such treatments are intended to cover, since what is needed is a (reasonably) accurate calculation (or estimate) of diffraction at large angles to the incident beam propagation direction. Therefore, a calculation using the fundamental principles of wave behavior is preferable to attempts to use small-angle approximations under large-angle conditions. Elementary application of Huygens's construction shows that this approach readily demonstrates the possibility of diffraction towards the microwave receivers from a Gaussian beam.

## ESTIMATE OF PERPENDICULAR DIFFRACTION, EXTERNAL REFLECTORS

In principle, diffraction theory may be used to predict the propagation of waves in free space, as this is the limit of the problem of diffraction from an aperture that is made larger and larger until it vanishes. In the idealized Li-Baker detector with external reflectors (Stephenson, 2008) examined here, there are no physical obstructions in the GB propagation path but the very fact that the beam intensity drops off away from the *z*-axis is equivalent to obstructing the wave by an aperture of transparency varying as a function of the radius from the $z$-axis. Therefore, in principle the use of diffraction theory is appropriate to estimate the power that may appear in the BPF masquerading as signal PPF. In a practical Li-Baker detector the situation is more serious, because it is likely that the GB will be generated within a constrained volume (the interaction volume) which must be lined with microwave absorbent material to prevent unwanted scattering of microwave power. The cross-section of this volume constitutes a diffraction "aperture" since the GB itself has strictly infinite extent. Nevertheless, by subtle design, this may be made to approximate the ideal case of a Gaussian microwave beam which is examined here. To estimate how much power will be diffracted into the $x$ direction from a $z$-directed Gaussian beam, start by writing the oscillatory electric field amplitude on-axis as

$$E_o = H_o \eta, \tag{1}$$

where $\eta$ is the wave impedance in the medium (presumably a vacuum) and $H_o$ is the oscillatory magnetic field amplitude. The peak value Poynting vector for the Gaussian beam on-axis is

$$S_z = E_o H_o = H_o^2 \eta = 2I_z \tag{2}$$

where $I_z$ is the measured power per unit area in the $z$-direction (and the factor 2 is introduced to convert from peak value to an average over peaks and zeros of the Poynting vector). Therefore, the magnetic field amplitude is given by

$$H_o = \sqrt{(2I_z/\eta)} \tag{3}$$

on axis, and since the beam has a Gaussian profile the value of magnetic field amplitude off-axis is given by

$$H_{GB} = \sqrt{\frac{2I_z}{\eta}} \exp\left(-\frac{x^2+y^2}{w^2}\right), \tag{4}$$

where $w$ is the beam radius. To estimate the magnitude of diffracted rays, the classical Fresnel-Kirchhoff diffraction formula (Pedrotti, Pedrotti, and Pedrotti, 2007) is a good starting point as it has a clear physical interpretation and so can readily be adapted to the present case. In original form this applies to spherical wavefronts extending over a restricted area. Here it is required instead to apply to a plane wave of Gaussian profile, and it is employed here to estimate the amplitude of a plane wave propagating perpendicularly as a result of diffraction. (Note that Sheppard and Hrynevych (1992) also rewrote this formula for the case of a plane wave in their Eqn. (3) but their expression is clearly erroneous as it omits the incident amplitude.) This gives the oscillatory magnetic field for a diffracted wave propagating in the $x$ direction as

$$H = -\frac{jk}{2\pi}\sqrt{\frac{2I_z}{\eta}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tfrac{1}{2}\exp\left(-\frac{x^2+y^2}{w^2}\right)\frac{\exp(jkx)}{r}\,\mathrm{d}x\mathrm{d}y\,, \tag{5}$$

where the factor $\exp(jkx)$ accounts for the optical path length, $r$ is the radial distance from the observing point (typically a microwave receiver in the present application) to the point $(x,y,0)$, and the ½ is the Fresnel Obliquity factor for $\theta = 90°$, i.e., diffraction perpendicular to the propagation direction (Pedrotti, Pedrotti, and Pedrotti, 2007). Also, the total electric field at large $x$ is given by

$$E = H\eta = -\frac{jk}{2\pi}\sqrt{2I_z\eta} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tfrac{1}{2}\exp\left(-\frac{x^2+y^2}{w^2}\right)\frac{\exp(jkx)}{r}\,\mathrm{d}x\mathrm{d}y\,. \tag{6}$$

At this point, the effect of GB polarization requires examination. In fact, the Fresnel-Kirchhoff treatment itself does not handle polarization entirely consistently (Pedrotti, Pedrotti, and Pedrotti, 2007). If the polarization of the GB is such that either its oscillatory magnetic or electric field is precisely in the $x$-direction, then it cannot support any diffracted wave in the $x$-direction. On the other hand, the EM field parallel to the GB propagation direction certainly will accumulate approximately according to the modified Fresnel-Kirchhoff formula of either Eqn. (5) or Eqn. (6). The problem basically reduces to that of estimating how much microwave power will diffract from an imperfect GB in practice. Clearly a wise designer of a Li-Baker detector will choose the GB polarization that minimizes diffraction in the same direction as the required signal PPF, but a slightly different approach is needed to estimate the diffraction intensity theoretically.

Suppose initially that the GB polarization used has one of its oscillatory EM fields within the $x$-$y$ plane angled at 45° to $x$ (and also at 45° to $y$). This is equivalent to a superposition of two resolved component waves: the first, oriented with its $x$-$y$ plane oscillatory field parallel to $y$ or at 90° to $x$, and the second, oriented with its $x$-$y$ plane oscillatory field parallel to $x$. In the perfect case, the wave oriented with its $x$-$y$ plane oscillatory field parallel to $x$ produces no diffraction directed towards the $x$-axis. However, the wave oriented with its $x$-$y$ plane oscillatory field at 90° to $x$ is oriented precisely to give maximum diffraction towards the $x$-axis. Moreover, the resolved amplitude of this wave is √½ times the amplitude of the original wave oriented with its $x$-$y$ plane oscillatory field at 45° to $x$. This introduces an extra factor of √½ in the field quantities, or ½ in the diffracted power calculated from Eqn. (6).

This is clearly not the ideal orientation of GB polarization for minimizing $x$-axis diffraction. However, using this geometry allows a theoretical estimate to be made of the likely diffraction power in this case. Then, having made this estimate, the result can later be reduced by a factor estimating the likely precision of the cancellation of diffraction power in the case where the closest practical approximation to the ideal orientation is used.

The radial distance $r$ in the denominator of Eqns. (5) and (6) has traditionally caused many problems in evaluating diffraction amplitudes, and many approximations have been devised to make analytic progress. Here, in the interests of obtaining an order-of-magnitude estimate of the diffraction amplitude, an approximation analogous to that of Fresnel (Sheppard and Hrynevych, 1992) is used, by assuming that $r$ is sufficiently constant over the region of greatest contribution to the integral that it may be taken outside the integral. Fresnel made this approximation for small diffraction angles, but it is also a reasonable approximation in the present case, since the microwave receivers are typically expected to be located at $x = \pm 1$m away from the GB whereas the GB extends only over a radius of around 3cm (Baker, Stephenson and Li, 2008). So, the peak value Poynting vector in the $x$-direction is given by

$$S_x = \tfrac{1}{2}\left|EH\right| = \tfrac{1}{2}\left|H^2\eta\right| = \tfrac{1}{2}\frac{k^2}{4\pi^2}\frac{2I_z}{4r^2}\left|\int_{-\infty}^{\infty}\exp\left(-\frac{x^2}{w^2}\right)\exp(jkx)\mathrm{d}x\right|^2\left|\int_{-\infty}^{\infty}\exp\left(-\frac{y^2}{w^2}\right)\mathrm{d}y\right|^2 \qquad (7)$$

where the ½ introduced is the polarization correction, so that the ratio of observed intensities between $x$-axis and $z$-axis is

$$\frac{I_x}{I_z} = \frac{S_x}{S_z} = \frac{k^2}{32\pi^2 r^2}\left|\int_{-\infty}^{\infty}\exp\left(jkx-\frac{x^2}{w^2}\right)\mathrm{d}x\right|^2\left|\int_{-\infty}^{\infty}\exp\left(-\frac{y^2}{w^2}\right)\mathrm{d}y\right|^2 \qquad (8)$$

or, by completing the square,

$$\frac{I_x}{I_z} = \frac{k^2}{32\pi^2 r^2}\left|\int_{-\infty}^{\infty}\exp\left[-\left(\frac{x}{w}-\frac{jkw}{2}\right)^2-\left(\frac{kw}{2}\right)^2\right]\mathrm{d}x\right|^2\times\left|\int_{-\infty}^{\infty}\exp\left(-\frac{y^2}{w^2}\right)\mathrm{d}y\right|^2. \qquad (9)$$

The integrals may be evaluated by making the substitutions

$$X = \frac{x}{w}-\frac{jkw}{2} \text{ and } Y = \frac{y}{w} \qquad (10)$$

giving the intensity ratio as

$$\frac{I_x}{I_z} = \frac{k^2}{32\pi^2 r^2}\left|\int_{-\infty-\frac{1}{2}jkw}^{\infty-\frac{1}{2}jkw}\exp(-X^2)\mathrm{d}X\right|^2 w^2\exp(-\tfrac{1}{2}k^2w^2)\left|\int_{-\infty}^{\infty}\exp(-Y^2)\mathrm{d}Y\right|^2 w^2. \qquad (11)$$

The second integral in this expression takes the well-known value $\sqrt{\pi}$. The first integral is a contour integral in the complex $X$ plane and, as the integrand has no poles in the complex $X$ plane, it can be split into three parts:

$$\int_{-\infty-\frac{1}{2}jkw}^{\infty-\frac{1}{2}jkw} \exp(-X^2)\mathrm{d}X = \int_{-\infty-\frac{1}{2}jkw}^{-\infty} \exp(-X^2)\mathrm{d}X + \int_{-\infty}^{\infty} \exp(-X^2)\mathrm{d}X + \int_{\infty}^{\infty-\frac{1}{2}jkw} \exp(-X^2)\mathrm{d}X . \tag{12}$$

The second of these contributions also takes the value $\sqrt{\pi}$, and because of the rapidly decaying value of $\exp(-X^2)$ as $|X| \to \infty$, the first and last contributions are zero, so that the intensity ratio is

$$\frac{I_x}{I_z} = \frac{k^2 w^4}{32 r^2} \exp(-\tfrac{1}{2} k^2 w^2) . \tag{13}$$

Note that because of the negative exponential, this expression tends to zero as $w$ increases, i.e., as the Gaussian profile metamorphoses into an infinite plane wave. Since this is a purely wavelike effect, the superimposed static magnetic flux density has no influence, and the diffraction pattern is actually radial rather than being directed purely along the $x$-axis.

## ESTIMATE OF DIFFRACTION FROM INTERNAL REFLECTOR

It has been proposed that placing a microwave reflector *within* the GB will prevent reflection of the perpendicular diffraction onto the microwave receivers (Baker, Stephenson and Li, 2008). It is true that such a reflector will be symmetrically immersed in the GB. However, the reflector is an obstruction in the GB path, and so diffraction from the reflector itself is likely to be serious problem in this configuration. The situation is illustrated in Fig. 3. For simplicity, a uniform incident beam is assumed, and the reflector is assumed planar, but neither of these assumptions will lead to a large error in estimating the diffraction intensity as the reflector must, for maximum sensitivity, sample the greatest intensity of the GB.

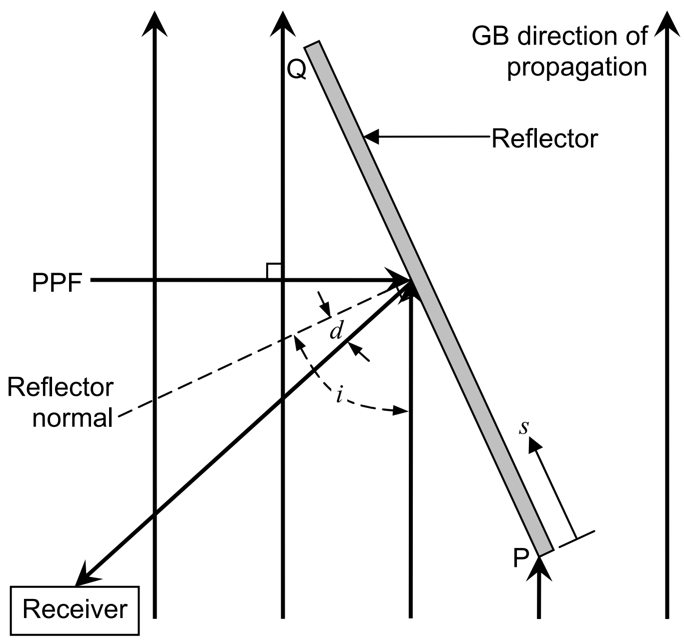


**FIGURE 3.** Diffraction From Reflector Embedded in Gaussian Beam.

In Fig. 3, the angle of incidence is $i$, and the angle of photon detection (and of diffraction) is $d$. The reflector is assumed rectangular and the length $s$ is measured along the reflector from corner P. Therefore, at distance $s$ measured along the reflector, the GB phase shift from incident to received parallel beams relative to $s = 0$ is given by $ks(\sin i + \sin d)$. All points on the reflector illuminated by the GB will act as Huygens secondary sources, and if the total path difference for the ray reflected at corner Q (the other end of the reflector) is an integer number of wavelengths then the diffracted amplitude to a first approximation is zero as all positive amplitude contributions are cancelled by negative contributions. Maximum diffraction power is given if the path difference from Q is a halfinteger number of wavelengths. The number of complete cycles contained in the path difference from Q compared to P is given by $ks(\sin i + \sin d)/(2\pi)$ where $s$ is the reflector length from P to Q, and the width of the obstruction is proportional to cos $i$. So, for a large reflector, the power multiplier of the beam in far field diffracted at angle $d$ to the normal is

$$\left| A \frac{\tfrac{1}{2}\cos i}{ks(\sin i + \sin d)/(2\pi)} \right|^2 = \left| \frac{1+\cos(180° - i + d)}{2} \times \frac{\pi \cos i}{ks(\sin i + \sin d)} \right|^2 = \left| \frac{\pi[1-\cos(i-d)]\cos i}{2ks(\sin i + \sin d)} \right|^2 , \tag{14}$$

where $A$ is the Fresnel obliquity factor again. Although this is the value corresponding to the local peak of the diffraction, in practice the received diffraction is unlikely to be significantly less because the microwave receiver itself is not a point aperture but receives a range of angles so that receiving truly zero diffracted power is impossible.

## DISCUSSION

A valid question is whether free-space perpendicular diffraction effects occur in the radiation from a radio pulsar that is many hundreds of light years distant, or from a conventional collimated microwave communications beam. In the former case, the radiation from a pulsar located at astronomical distance is not directed into a beam but follows the well-known Hertzian dipole radiation pattern with wide radiation lobes, the calculation of which itself takes account of diffraction effects from first principles. The received radiation intensity reduces according to the well-known $1/r^2$ law (which is the reason why the intense radiation transmitted is so weak when measured on the earth), which itself could even be interpreted as an extreme case of diffraction widening the beam. On the other hand, in the latter case, there is indeed diffraction from any directed electromagnetic radiation beam propagating in free space (as opposed to being contained in a waveguide such as an optical fiber, for which the propagation mechanisms are different). It is well-documented that a perfectly collimated optical laser beam directed at the moon has diverged to being several miles across by the time it hits the moon's surface (Espenak, 1994), which has mixed effects on experiments to measure the earth-moon distance by laser rangefinder (it reduces the signal amplitude but makes it easier to locate the reflector on the moon).

However, the perpendicular diffraction estimated in this paper is so small that generally in most applications it may safely be neglected. When the power in the diffracted rays is subtracted from the incident beam power, there remains virtually unchanged intensity. Nevertheless, this result could be important in the Li-Baker detector where tiny perturbations such as this might be significant.

Considering firstly the design with external reflectors (Stephenson, 2008), typical parameter values are that the wavenumber is $k = 2\pi f/c = 209$rad/m at 10GHz, and the receiver distance is $r = 1$m. The smallest GB radius that has been proposed is $w = 0.03$m (for an interaction volume of 6cm diameter, a little larger than the minimum diffraction-limited spot size $w = 2/k$), which gives the estimated intensity ratio of $x$-directed diffraction to $z$-directed GB calculated from Eqn. (13) as $3\times10^{-12}$. If the polarization is such that with perfect alignment there should be identically zero diffraction, no practical microwave system could be expected to do better than cancellation to 1%, giving an intensity ratio around $3\times10^{-14}$. Fig. 4 shows a graph of the ratio $I_x/I_z$ calculated from Eqn. (13) for a range of other possible values of beam radius $w$ using the same parameter values $k$ and $r$, but excluding the empirical correction for polarization cancellation which is easily imposed.

The external reflectors will be used to focus the PPF onto the microwave receivers. These reflectors define the interception aperture of the receiver system, irrespective of the actual receiver input aperture, and the number of BPF photons captured will be the product of the BPF multiplied by that interception aperture area. Most previous analyses (Baker, Stephenson and Li, 2008) have assumed that the interception aperture area is equal to the GB cross-sectional area. This means that no scaling of the BPF results according to the area through which the BPF is received is necessary for the present comparison with previous work (but if the interception aperture is changed then the received BPF power will be proportional to its area). If the GB power is 1000W, 10GHz photons each have energy $0.66\times10^{-23}$J, so there are $1.5\times10^{26}$ photons produced per second in the GB over the cross-sectional area of the interaction volume (circular, of diameter roughly 6cm). Hence, for the smallest value of beam radius proposed ($w = 0.03$m), the number of GB photons diffracted into the microwave receivers (the BPF) is $4.5\times10^{12}$ per second, assuming the best estimate of the intensity ratio (corresponding to the GB polarization choice for minimizing perpendicular diffraction) from the previous paragraph. On integration over 1000s, that corresponds to $4.5\times10^{15}$ photons, which will have a shot noise component corresponding to $\sqrt{(4.5\times10^{15})} \approx 7\times10^{7}$ photons randomized.

Baker, Woods and Li (2006) noted that for a HFGW amplitude of $3\times10^{-32}$ and a continuous GB power of 10W at 4.9GHz, a PPF of 490 photons would be expected in a typical Li-Baker detector design when summed over 1000s. Scaling to a power 1000W and frequency 10GHz, this corresponds to 24010 PPF photons. There appears to be no possibility of finding 24010 PPF photons in $7\times10^{7}$ photons of shot noise; this is a signal-to-noise ratio of –35dB.

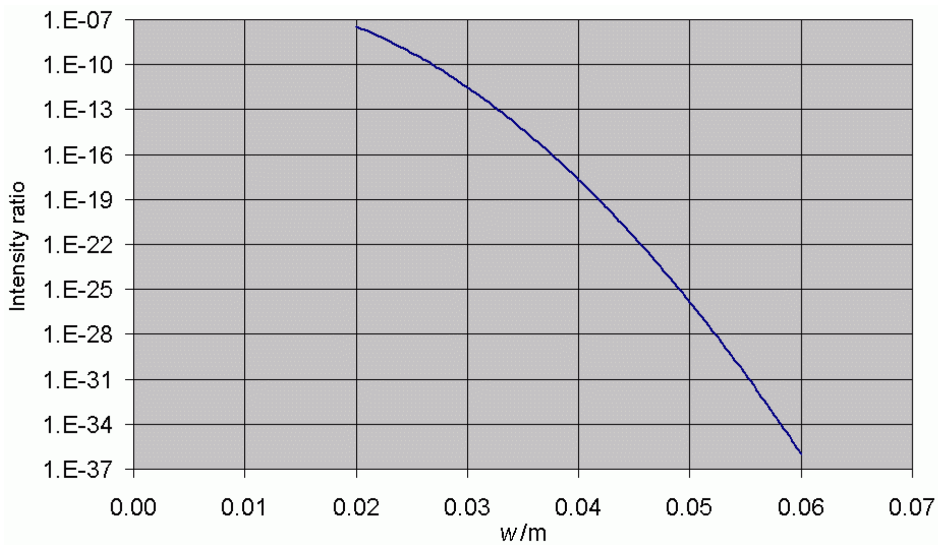


**FIGURE 4.** Perpendicular Diffracted Intensity Ratio $I_x/I_z$ (Note: Logarithmic Scale) from Eqn. (13), for 10GHz GB at receiver distance $r$ = 1m.

This situation may be improved by capitalizing on the very fast exponential (squared) variation of the diffracted intensity with $w$ and/or with $k$ as illustrated in Fig. 4. Increasing $w$ to 0.05m at 10GHz gives an intensity ratio of $1.3\times10^{-26}$, even if not reduced at all by polarization considerations. This gives 2000 photons in 1000s in the BPF focused to the microwave receivers, corresponding to a shot noise of around 44 photons over 1000s. Note that the PPF intercepted is also slightly reduced because the GB intensity is reduced, but not by the same factor as the noise, so that the signal-to-noise ratio will now be acceptable. Therefore, at the same frequency, the GB must be rather wider than the narrowest envisaged. However, this estimate still requires trusting the GB geometry to very high precision. The high sensitivity of the BPF to variations in $w$ means that this parameter will have to be chosen carefully to assure adequate performance, in case the estimates here are too optimistic.

In deriving this result, it has been assumed that the incident GB is polarized in such a way as to minimize the diffracted power, but that since this essentially relies upon precise alignment of the $x$-axis and one of the electromagnetic fields of the GB over the entire interaction volume then inevitably this cancellation is not exact but depends upon the precision to which the GB can be generated, transmitted, and propagated through the interaction volume. An arbitrary practical estimate of 1% precision was assumed possible in an actual microwave system. It follows that one way to reduce the diffraction in practice is therefore to improve the precision of the microwave system. This, however, is far from easy, and the law of diminishing returns applies (expenditure rises rapidly out of proportion to the improvement gained).

The question of the accuracy of the result obtained must also be addressed. The Fresnel-Kirchhoff formula is an approximation, and its use assumes that the source and receiver distances are much larger than the extent of the incident beam aperture, and in addition that this incident beam aperture is itself much larger than the wavelength (Pedrotti, Pedrotti, and Pedrotti, 2007). It is also expected that perfect combination of secondary waves occurs. It is well-established that free space is a linear medium for EM wave propagation, and the interaction and detection volumes in any practical Li-Baker detector will be under high vacuum to prevent water condensation at the cryogenic temperatures necessary for operation of low-noise microwave receivers, so that this assumption appears reasonable. However, more importantly, when large amplitude waves are summed algebraically, their waveforms must be accurately known for the idealized summations used here to reflect accurately what will occur in practice.

The GB will almost certainly not be accurately Gaussian in form, and it is likely that therefore the results here will be affected by these considerations.

The parabolic reflectors for the wanted PPF will be employed to reflect and focus the PPF onto microwave receivers located below the horizontal plane of the interaction volume. However, the reflectors also reflect and focus unwanted diffraction BPF originating in the same direction as the PPF, so that this arrangement cannot solve the basic problem of distinguishing between BPF and PPF propagating parallel to each other. It is, of course, a basic requirement that the walls of the interaction and detection volumes will be excellent absorbers so that none of the BPF photons diffracted in any direction can reflect onto the microwave receivers. This imposes quite strict requirements on the performance of the absorber to be used.

The configuration with an internal reflector performs much worse than this. Assuming the same operating frequency as before, for a typical proposed Li-Baker detector design (Baker, Stephenson and Li, 2008), the incident angle may be around $i = 60°$, the detection angle $d = 90° - i = 30°$, and the length of the reflector $s = 0.1$m, giving a diffracted power ratio of $1.4\times10^{-5}$. This will be reduced a further 10 times typically to around $1.4\times10^{-6}$ because of the reduced aperture area of the microwave receiver compared to the reflector (for the purposes of making a conservative estimate, focusing of the diffracted photons is assumed not to occur). The diffraction from the reflector must therefore be reduced by a factor around $1.3\times10^{15}$ times for adequate performance. Even if the estimate made here of the diffracted microwave power were considerably in error, the improvement needed appears prodigious. Suppose, for example, that the present estimate were too large by a factor $10^9$ (implying gross underestimate of the cancellation of diffraction possible in practice); then, the microwave system must still be improved by a precision of more than a further *one million times* for HFRGW detection. Changing the incident angle to grazing incidence, $i = 89°$, corresponding to $d = 1°$, reduces the power ratio only to $4\times10^{-7}$. Splitting the single reflector into a series of slats is one possible method of reducing this diffraction, and is the subject of ongoing investigation, but a reduction of such a great magnitude seems unlikely. At present, the detector design with external reflectors appears more promising.

If there are 24010 PPF photons after integrating over 1000s, the shot noise must be less than ~12000 photons in 1000s for efficient detection. This is equivalent to a BPF of fewer than $1.5\times10^8$ photons in 1000s. In 1000s there are $1.5\times10^{29}$ photons in the GB, so it is required that the GB spillover rate to the microwave receivers shall be less than 1 in $10^{21}$. It seems unlikely that this could be obtained in practice without considerable care in design, particularly regarding direct EM breakthrough due to non-ideality of screening components, even if the estimates given above are accurate. Therefore, the conclusion is that efficient BPF screening of some kind is needed between the GB transmitter and the receivers, in addition to reliance on the directionality of the GB. Whether such screening can be as efficient as required here, while nevertheless allowing the PPF to be detected, is still open to question, given the extreme levels required in this application.

## CONCLUSIONS

The spillover from the Gaussian beam in previously published envisaged implementations of the Li-Baker detector is several orders of magnitude greater than can be tolerated if the detector is to be capable of detecting and investigating the cosmic high-frequency relic GW background radiation. The Li-Baker detector must be designed in such a way that the diffraction reaching the microwave receivers is reduced as far as possible by employing a suitable geometry and highly absorbent walls for the interaction volume. The configuration with a reflector internal to the GB performs very poorly in this regard. The configuration with external reflectors may perform acceptably if the diameter of the interaction volume is increased, though in view of the extreme levels of spillover prevention required it may also be necessary to utilize efficient screening as well if this configuration is to be realizable as a useful detector of relic HFGW.

## NOMENCLATURE

$A$ = obliquity factor
$\mathbf{B}$ = static magnetic flux density (T)
$d$ = angle of diffraction (°)
$E$ = oscillatory electric field (V m$^{-1}$)
$H$ = oscillatory magnetic field (A m$^{-1}$)
$i$ = angle of incidence (°)

$I$ = intensity (W m$^{-2}$)
$j$ = √(–1)
$k$ = wavenumber (rad m$^{-1}$)
$r$ = distance from observing point (m)
$d$ = angle of diffraction (°)
$s$ = length measured along reflector (m)
$w$ = Gaussian beam radius (m)
$x$ = coordinate (m)
$X$ = dummy integration variable
$y$ = coordinate (m)
$Y$ = dummy integration variable
$z$ = coordinate (m)
$\eta$ = wave impedance (Ω)
$\theta$ = angle to incident wavevector (°)


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